# Designing cable-stayed bridges with Genetic Algorithms

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**Abstract.** Cable-stayed bridges construction involves the determination of a high number of design variables, both static and dynamic. Moreover, the properties of such variables make them statically indeterminate, meaning that a change in one design variable affects the response of the entire structure. This property makes the design of cable-stayed bridges a complex optimization problem. In this work, we use a Genetic Algorithm to evolve solutions for this problem. A set of experiments are executed, where conventional variation operators are used for exploring the solution space. The first experiments suggest that this is a problem with a deceptive landscape. However, we show that we can design solutions that optimize structural objectives. Moreover, we want also to minimize costs while presenting different optimized solutions. In the second set of experiments, we included a baseline solution in the population to evaluate if we could find better solutions using this approach. The results on the second set showed that it was possible, thus we moved to the third set of experiments with more parameter tuning. The experimental results suggest that we are able to find new and suitable solutions for the problem comparable to the existing baseline approach.

**Keywords:** Genetic Algorithm, Bridge Design, Optimal Design, Deceptive Landscape, Structural Dynamics

## 1 Introduction

Structural Engineering is a fertile ground for new ideas, structural systems, concepts, in particular when designing complex structures [1]. Usually, the work of structural engineers is performed as a series of 'trial and error' iterations, where changes are made to a computer model of the structure until the structure is stable and satisfies standard requirements presented in the Eurocodes – the structural design codes for the European Union, in this case, EN 1991 to EN 1998 and Setra Footbridge guidelines. This process is strongly dependent on each designer capability, creativity and experience.

Structures can be roughly divided into two groups: statically determinate and indeterminate structures [2]. Statically determinate structures are easier to solve and, for a fixed geometry, the stresses in all internal elements are known. Statically indeterminate structures become more complex because a change in a particular structural element changes the stress distribution in the entire structure, even without changing the geometry. A complex statically determinate structure can be calculated by hand in a reasonable amount of time, while an indeterminate structure with moderate complexity cannot. In recent years, the advance in computers using the finite element method allows engineers to analyse even complex, indeterminate structures in a reasonable amount of time [3].

Artificial Intelligence (AI) has long been identified with high potential to improve the work of structural engineers [4]. Moreover, Evolutionary computation has been used for solving structural design problems [5, 6]. Other AI methods have been proposed to tackle specific problems such as pattern recognition and machine learning for damage detection and structural health monitoring [7].

Research works on the optimum design of Cable Stayed Bridges (CSBs) have increased over the past three decades. Most of this can be attributed to the increase in computational performance for the analysis of the structure as well as CSB construction technology [8, 9]. These two key factors contributed, for example, to the increase of the CSB record span from 490m in 1991 (Ikuchi Bridge in Japan) to 1104m in 2012 (Russky Bridge in Russia). The same trend does not happen to other types of structures. For example, suspension bridges where the record span of 1991m dates back to 1998 (Akashi Kaikyo Bridge in Japan), or arch bridges where the current record span of 552m in 2009 (Chaotianmen Bridge in China) is not much longer than older structures such as the Bayonne Bridge opened in 1931 in the USA with 510m or the famous Sydney Harbour Bridge opened in 1931 with 503m. The reason for this is that it was not possible to calculate recent CSB without the advances in computer science since they are highly nonlinear and static indeterminate structures. Even in 1999, structural engineers did not have the necessary computational capabilities to solve all load cases in the most complex cable-stayed bridges [8]. Evolutionary computation was not possible back then as a single analysis of the structure was very time-consuming.

The optimum design of CSB can be viewed as a relevant benchmark for artificial intelligence algorithms as it requires to consider both the geometry, cross-section, cable tensioning and control devices to address the real-world problems. Other types of structures such as arch bridges require mostly to compute the section sizing [10].

In terms of contributions we enumerate the following: (i) an out-of-the-shelf evolutionary approach to optimise and explore different solutions for a controlled cable-stayed bridge problem; (ii) different fitness functions designs based on various criteria and objectives of the problem; (iii) comparison between a set of optimisation approaches proposed on this work and; (iv) the obtained results suggest that the system dynamic properties create a deceptive landscape that must be adequately addressed in order for the algorithm to find efficient solutions.

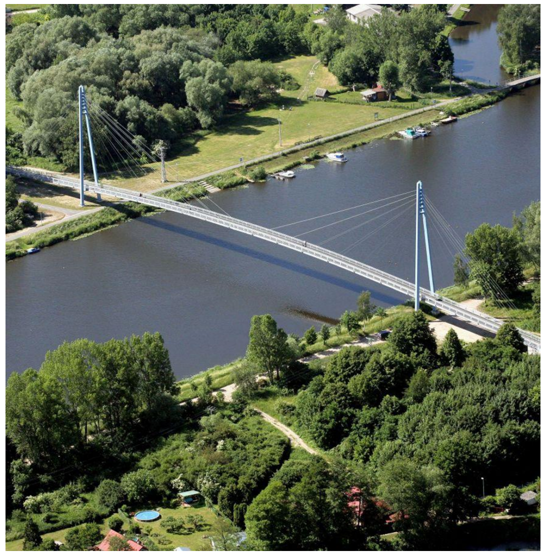

**Fig. 1.** An example of a CSB bridge.

The remainder of the document is organised as follows. In Section 2, we overview the related work regarding optimising the design of bridges. Next, Section 3, we explain the approach of this work for modelling this problem and system dynamics with a traditional Genetic Algorithm (GA). In Section 4, we present the set of experiments performed around this case study. Afterwards we present the results in Section 5 and we draw our final conclusions in section 6.

## 2 Related Work

The previous research works on the optimum design of CSB started by addressing the cable tensioning problem with fixed geometry and structural sections [11–13]. This problem has also been addressed more recently using genetic algorithms (GA) [14, 15].

The simultaneous optimization of the geometry, sizing and cable tensioning has also been studied using gradient-based optimization techniques [16, 17] and more recently with genetic algorithms [18] with simpler modelling constraints, conditions and with a smaller number of variables than the work presented in this document. Overall, based on the literature, the performance of gradient-based optimization using multi-start procedure is faster than using GA. However, the main drawback is that it requires a very high amount of time to program the details and sensitivities of the problem, unlike GA that it is a more hands-off approach, i.e. that can be used with much lower implementation effort.

Another drawback is that if one changes the structural analysis software, the sensitivity-based code will no longer be available, while the GA can be more easily adapted to another software.

The inclusion of the dynamic loads creates additional constraints for the design problem. Previous researches have focused on earthquakes [19, 20], wind aerodynamics [21–23] and pedestrian induced action in cable stayed footbridges [24–26].

Both wind aerodynamics and pedestrian induced action cause resonance in the structure, which magnitudes are strongly dependent on the structural damping ($\xi$) by a factor of ($1/2\xi$). Usually, steel footbridges are vibration prone since they exhibit very low damping (0.4%) and thus the dynamic criteria are a governing factor for the design. There are mainly three options to mitigate the vibrations of footbridges: (i) increase the mass of the structure. This is not efficient or desirable since footbridges are required to be slender and aesthetically appealing to become part of the landmark; (ii) Changing the bridge dynamic properties such that the vibration frequency of each mode does not fall within the critical range for the pedestrian induced actions; (iii) Inclusion of control devices such as viscous dampers or tuned mass dampers (TMDs) which was the option to retrofit the London Millennium Footbridge, for example, [27, 28].

To the author's knowledge, no previous work addressed the optimum design problem of controlled pedestrian CSB using evolutionary computation algorithms. The problem is based on previous works by [25, 26] was a sensitivity based algorithm is used to find the optimum results for both geometry, section sizing, cable tensioning and control device properties. This has the advantage of having already a feasible and improved solution (not necessarily a global optimum) to compare the results with. Also, the sensitivity based algorithm requires the researchers to employ the sensitivities analysis, the derivation of the equilibrium equations, which constitute a high time expenditure in the implementation of the optimization algorithm. We are aware that the nature of the problem can call for different approaches, including multi-objective optimization, nevertheless we intend to explore and harness the complexity of the problem using a standard GA.

## 3 The Approach

We use a standard Genetic Algorithm [29] to search for solutions in the search space. The individuals are a set of Design Variable (DV)s that parameterize the design of the bridge. To that effect the following fixed DVs have been considered:

- Bridge Length: LTotal=220m;
- Bridge Width : 4m;
- Tower Height below deck: 10m;
- Number of cables (Ncables)=4

The number of cables is considered for each side of each tower. The structure is parameterized with four towers leading to a total of 8 cables for the entire structure. An example of such a structure is depicted in figure 1.

**Table 1.** Design variables description.

| | Description |
|---|---|
| Geometry | |
| DV1 | Central span (tower to tower distance) of the structure |
| DV2 | Distance between the first and second cables anchorage in the lateral span of the deck |
| DV3 | Distance between the tower and the first cable in the central span |
| DV4 | Distance between the last cable anchorage and the bridge symmetry axis |
| DV5 | Height of the towers |
| DV6 | Distance where the cables are distributed in the top of the towers |
| DV7 | Distance between the top of each tower |
| DV8 | Distance between each tower at the base |
| Control | |
| DV9 | Transversal stiffness of the tower-deck connection |
| DV10 | Vertical stiffness of the tower-deck connection |
| DV11 | Transversal damping of the tower-deck connection |
| DV12 | Vertical damping of the tower-deck connection |
| Sectional and tensioning | |
| DV13 | Added mass of the concrete slab |
| DV14 | Deck section |
| DV15 | Deck section (triangular section) |
| DV16-19 | Tower sections (rectangular hollow section) above and below the deck |
| DV20 | Cables pre-stress |
| DV21 | Cables cross section |

The DVs have been considered similar to works available in the literature, nonetheless, they were normalized. The objective of this normalization is for future works to obtain a database of solutions that could be more easily correlated even for different bridges. A more detailed explanation about the DVs can be found in Ferreira et al. work [25]. Each specific DV affect a specific geometric, sectional or control property of the bridge as shown in table 1.

The implementation is a standard GA, where a fixed-sized population of solutions is evaluated, selected and submitted to variation operators in a loop with a certain termination criterion (i.e., a predetermined number of generations). The representation of an individual of the population consists of an array of floats representing the values of the DVs. An aspect of the representation of the individuals is that each DV has its domain of range values, which are presented in table 2. The selection implemented is the tournament selection where the best N individuals are selected to apply variation operators. The conventional variation operators, crossover and mutation are applied. The crossover of this approach is the uniform crossover, where each DV has a chance to swap between the par-

**Table 2.** Individual's domain values.

| | Domain |
|---|---|
| Geometry | |
| DV1 | $[0.9, 1.2]$ |
| DV2 | $[0.7, 1.3]$ |
| DV3 | $[0.7, 1.3]$ |
| DV4 | $[0.7, 1.3]$ |
| DV5 | $[0.1, 2.0]$ |
| DV6 | $[0.1, 4.0]$ |
| DV7 | $[0.1, 1.3]$ |
| DV8 | $[0.1, 1.13]$ |
| Control | |
| DV9 | $[0.001, 1000]$ |
| DV10 | $[0.001, 1000]$ |
| DV11 | $[0.001, 1000]$ |
| DV12 | $[0.001, 1000]$ |
| Sectional and tensioning | |
| DV13 | $[0.1, 7.0]$ |
| DV14 | $[0.1, 80.0]$ |
| DV15 | $[0.5, 1.3]$ |
| DV16 | $[0.4, 1.5]$ |
| DV17 | $[0.1, 20.0]$ |
| DV18 | $[0.3, 20.0]$ |
| DV19 | $[0.3, 9.0]$ |
| DV20 | $[0.7, 3.0]$ |
| DV21 | $[0.5, 9.0]$ |

ents. The mutation is the replacement of a DV value for another drawn from a uniform distribution in the domain of the corresponding DV.

We are designing variables for the CSB, a task normally done by structural engineers. The objectives in a real-world scenario are to minimize the costs of constructions while maintaining the structural and safety constraints. Thus, we consider the following objectives for our approach: minimize the bridge costs and minimizing the structural constraints. These two objectives are affected by the chosen DVs. In order to complete the modelling of the GA, we need to design a proper fitness assignment that guides towards the solution of the problem. In eq. 1 is depicted as the fitness function used for the set of experiments of this work.

$$f(x) = \begin{cases} 1/S(x), & \text{if } S(x) > 1.04 \\ a + b/C(x), & \text{if } S(x) \leq 1.04 \end{cases} \tag{1}$$

Where x is an individual, C(x) is a function that computes the cost of an individual, S(x) is a function that evaluates the structural constraints of and individual, "$a$" is a bonus value constant for the "$S(x) > 1.00$" constraint and "$b$" is a constant for the target cost. The $C(x)$ is used to calculate the cost of the bridge

**Table 3.** GA's Parameters for *exp1* and *exp2*.

| Parameter | Setting |
|---|---|
| Number of generations | 5000 |
| Population size | 50 |
| Elite size | 1 |
| Tournament size | 2 |
| Crossover operator | uniform crossover |
| Crossover rate | 0.8 |
| Mutation operator | gene replacement |
| Mutation rate per gene | 0.1 |
| $a$ fitness constant | 50 |
| $b$ fitness constant | 0.9 |

according to some pre-determined pricing of the materials. The $S(x)$ calculates all security and structural constraints and returns the maximum value of each variable considered. From a practical perspective, we have only to ensure that the maximum value of $S(x)$ is bellowed 1.04, preferably, around 1.00. These two functions represent the two objectives that we must optimize, as minimization problems. Note that both, from a structural engineering perspective, are correlated. By increasing some of the values of the DVs we are strengthening the structure of the bridge which results in a low S(x) value but could translate to a costly solution, i.e., a high value of C(x).

## 4 Experimental Setup

We developed three experiments for this case study. The first experiment referred to as *exp1*, are the first set of experiments where we aim to optimize the problem further than a baseline conventional approach from [25]. Based on the results of *exp1*, later explained in Section 5, we performed experiments in which we insert in the starting population an individual of the best baseline approach. After drawing some conclusions from *exp2* we moved to some adaptations on the hyper-parameters and performed the experiments of *exp3*.

Note that, as said in 3, for this problem we had some fixed constraints and objectives. However, the approach can be adapted and some of these constraints can be included in the set of DVs. We fixed some of these constraints such as, the size of the bridge to 220 meters and the number of cables to 4, to compare with a baseline conventional solution which has the following optimized solution:

- Bridge cost: 91.354 k€
- Max Structural Constraints value: 0.9962

The evolutionary engine settings are presented in table 3. For *exp3* we changed the "Tournament size" to 3, the "Mutation rate per gene" to 0.2 and the "Crossover rate" to 0.85. Due to the stochastic nature of the approach, we performed 15 evolutionary runs for each experiment.

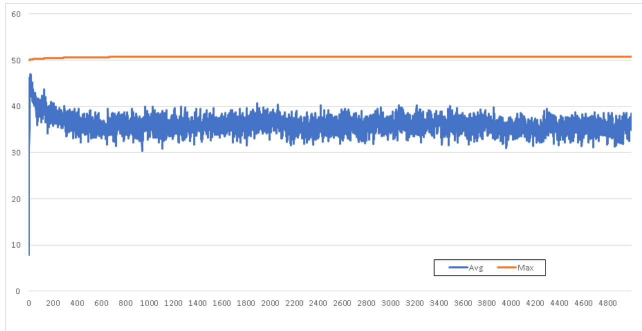


**Fig. 2.** The Average and Maximum fitness across generations for *exp1*. The results are averages of 15 evolutionary runs.

## 5 Experimental Results

For *exp1* we evaluate the approach in terms of fitness optimization, cost values and structural constraints. In figure 2 we can observe that we can optimize the fitness function. However, while looking at the average population, we see instability on the curve behaviour. After the first few 200 we have a lot of ups and downs from one generation to the other, around the same average value. This fast growth and the average population values oscillation suggest that the fitness landscape is deceptive. Fully investigating this, is out of the scope of the paper but it is an avenue of research that is already being pursued. Regarding the best solutions per generation, we can observe that rapidly grows and stabilizes. To further see the variations, we analyse the cost and structural constraints for the best individuals.

In terms of structural constraints, part of the first objective of the fitness function, we can observe on figure 3 that is achievable and maintained. It starts with lower values, which is normal but there is a small correlation between having this value low and the cost of the solution high, showing that we must find a solution that balances the two objectives.

In figure 4 we can observe the cost of the best individual across the generations. As expected, inversely like the fitness, the values rapidly decrease although in the first 100 iterations it is quite high. To see the results with more detailed we also provide a "close-up" plot, which shows the values of the cost after 400 iterations as shown in figure 5. We can observe that there are some alterations along the iterations, but, for the number of evaluations defined in the setup, the best solution on average only reaches the $110k€$ which is higher than the baseline approach. Since we have done several seeds and arrived at this result, at the time we questioned if it was possible to get or improve on the baseline approach using this approach.

Thus, we then move to *exp2* where we use the GA but we include a copy of the baseline solution in the starting population, promoting local exploration of solutions. We can observe the fitness behaviour in figure 6. The results sug-

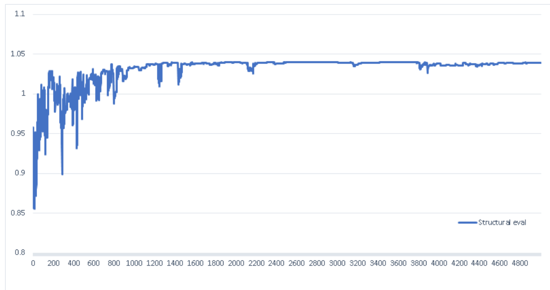


**Fig. 3.** The structural constraint value of the best individual across generations for *exp1*. The results are averages of 15 evolutionary runs.

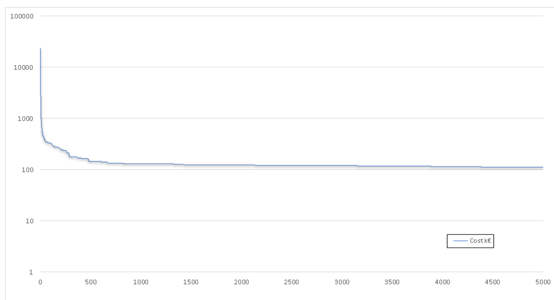


**Fig. 4.** The cost of the best individual across generations for *exp1*. The results are averages of 15 evolutionary runs.

gest that the behaviour as not changed. In terms of structural constraints we obverse little to none differences across generations in figure 7. The main point of conclusions here is the figure 8. It shows that we can further improve from the baseline solution, showing that there are other solutions for the problem. Furthermore, suggests that the parameters need to be adjusted for the problem at hand, something that motivated the final set of experiments of this work, the *exp3*.

After *exp1* and *exp2* we performed *exp3* knowing that it was possible to evolve solutions for the problem and it was possible to improve beyond the baseline solution. We performed some tests and determined the changes of parameters empirically. The rationale beyond the defined values is the following: we wanted to promote more changes from one iteration to the other, hence changing the increase on the mutation rate. We also increase the crossover rate and the size of the tournament to increase selective pressure.

A close up of the results after 400 iterations, in terms of cost, is presented in figure 9. We tend to have the same behaviour of *exp1* but now it surpasses the baseline approach under the same number of evaluations. We get values of cost around almost 90.00$k$€. In terms of structural, we have the values around 1.0


**Fig. 5.** The cost of the best individual across generations, starting from generation 400, for *exp1*. The results are averages of 15 evolutionary runs.

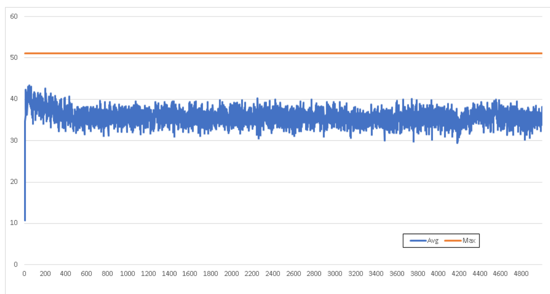


**Fig. 6.** The Average and Maximum fitness across generations for *exp2*. The results are averages of 15 evolutionary runs.

as shown in figure 10. The solutions evolved in this set of experiments are able of achieving the desired objective, while maintaining low costs.

We can observe in table 4 that the algorithm arrives at different solutions. Note that the results that are shown are from the best solutions found in each experiment and are just representative of the universe of solutions that the GA encounters. We can observe that some of the DVs are unchanged when compared to the baseline solution, which could be indicating that the best optimize solution only changes a few variables once we arrived at the baseline one. The best result observed during the set of three experiments is the a solution with the cost of : $90.893k€$ and structural constraints of 1.016. Overall the results show that we can find different and better solutions than the baseline solution obtained via gradient optimization.

Fig. 11 and 12 present the structural model geometry where each line represents a structural element. The Best *exp1* solution is awkward from a structural Engineering perspective at the first impression. The central span is lower than half of the bridge length (DV1¡1) which is not an usual CSB geometry. Also, the tower height to central span ratio (affected by DV5) is markedly low. The reason for this is that, in *exp1*, the GA could not balance the bridge, probably due to the lower mutation rate in this experiment. It then converged to a extradosed bridge

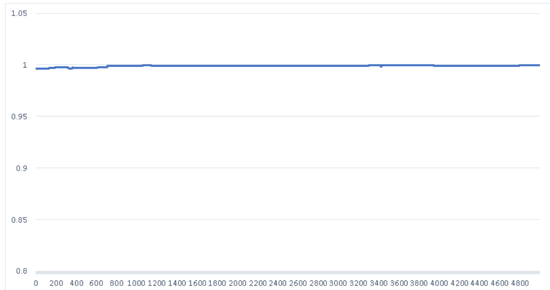


**Fig. 7.** The structural constraint value of the best individual across generations for *exp2*. The results are averages of 15 evolutionary runs.

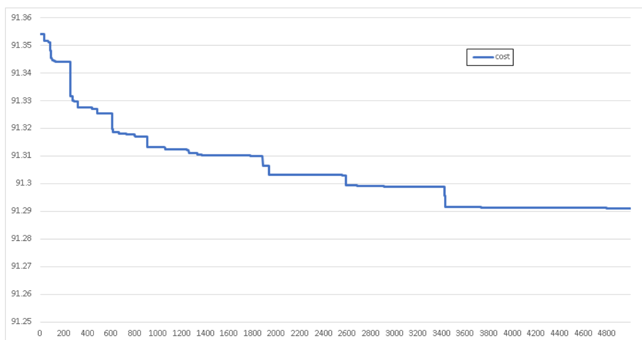


**Fig. 8.** The cost of the best individual across generations, starting from generation 400, for *exp2*. The results are averages of 15 evolutionary runs.

where the cable stay system becomes secondary and the bridge deck works as a continuous beam. This can also be seen in DV10 which has a very high value in Best *exp1*. In the other solutions the bridge floats over the tower intersection as only a very slender connection is required. the opposite happens in Best *exp1* where the tower-deck connection needs to be very rigid to support the bridge deck. This solution is not governed by the dynamic constraints, but rather by the static response, in particular, the deck stresses. As it can be seen in Figure 13, the response is well below the established limits. This is because the bridge is much stiffer than the baseline solution which also makes it more expensive.

The best solutions for *exp2* and exp *exp3* are very similar to the baseline. In terms of geometry the relevant difference is in the cable anchorage position that change in Best *exp2* and *exp3* (see Figure 12). Another main difference seems to be in DV10, which is the stiffness of the tower-deck connection in the vertical direction. Even if the numerical value is quite different the solutions have similar properties since the baseline stiffness is relatively low (this is why DV9 to DV12 have an exponential type domain as presented in Table 4). DV12 (damping in the vertical tower-deck connection) also presents a difference in the

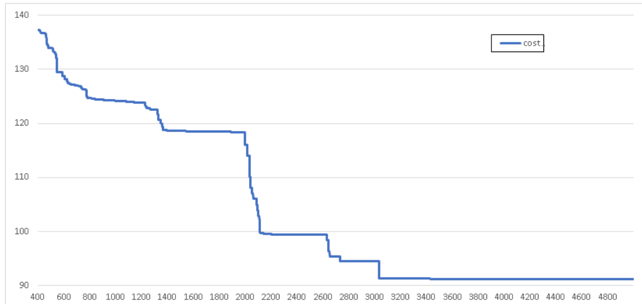


**Fig. 9.** The cost across generations, starting from generation 400, for *exp3*. The results are averages of 15 evolutionary runs.

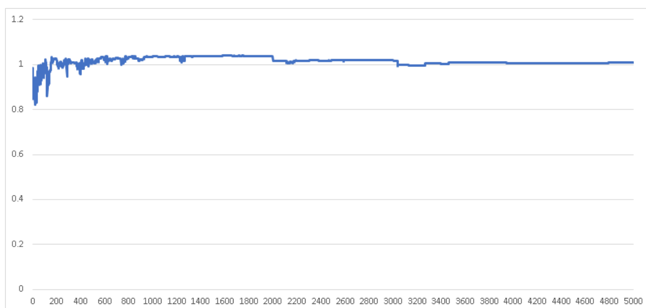


**Fig. 10.** The structural constraint value of the best individual across generations for *exp3*. The results are averages of 15 evolutionary runs.

results. The constraints have a sensitivity to this parameter. Figure 13 presents the deck peak accelerations in the vertical direction for three different solutions. The 1.018 S(x) value for the Best *exp2* solution can be viewed in this figure as the vertical acceleration slightly exceeds the limits.

## 6 Conclusions

We perform different experiments using a GA and compare them with a domain knowledge baseline solution. First, we show that a conventional Evolutionary approach is able to optimize the fitness function and find solutions that are: (i) suitable in terms of requirements; (ii) and that minimize the cost. Furthermore, the approach is able to find solutions that are suitable and comparable to the baseline solution.

The preliminary results using a conventional Genetic Algorithm to evolve towards the maximization of the structural objectives indicate that we are able to optimize the fitness function. However, while optimizing the structural objectives, the solutions found do not minimize the cost of solutions. When combining

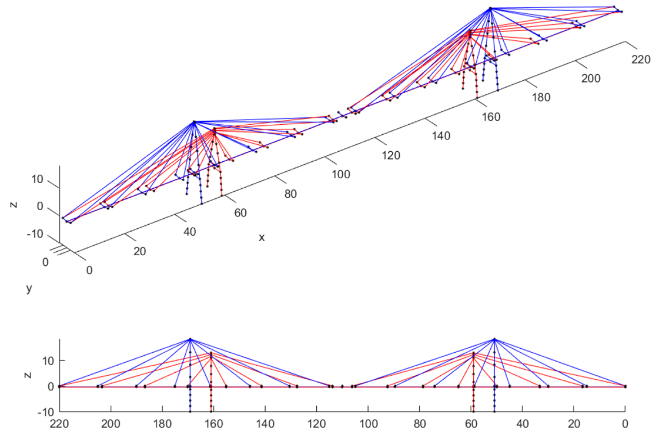


**Fig. 11.** Baseline (blue) vs Best Exp1 (red) structural model geometry of the bridge.

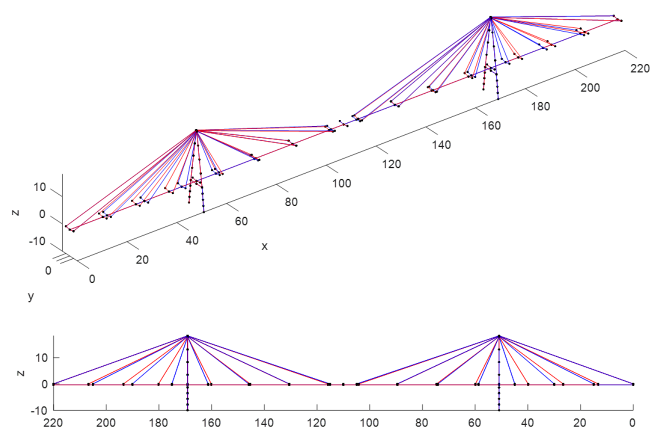


**Fig. 12.** Baseline vs Best Exp2=Best Exp3 structural model of the bridge.

**Table 4.** Best solutions of all the three experiments and the difference between cost, structural constraints and DVs values against the baseline approach.

| | Baseline(B) | dif(B,exp1) | Best exp1 | dif(B,exp2) | Best exp2 | dif(B,exp3) | Best exp3 |
|---|---|---|---|---|---|---|---|
| C(x) | 91.354 | -13.006 | 104.360 | 0.461 | 90.893 | 0.481 | 90.873 |
| S(x) | 0.996 | -0.043 | 1.039 | -0.022 | 1.018 | -0.021 | 1.017 |
| DV1 | 1.075 | 0.147 | 0.927 | 0.000 | 1.075 | 0.000 | 1.075 |
| DV2 | 0.700 | -0.224 | 0.924 | -0.598 | 1.298 | -0.599 | 1.299 |
| DV3 | 0.892 | 0.118 | 0.774 | -0.136 | 1.028 | -0.136 | 1.028 |
| DV4 | 1.300 | 0.001 | 1.299 | 0.000 | 1.300 | -0.009 | 1.309 |
| DV5 | 0.498 | 0.146 | 0.352 | 0.000 | 0.498 | 0.000 | 0.498 |
| DV6 | 0.100 | -0.717 | 0.817 | 0.000 | 0.100 | -0.002 | 0.102 |
| DV7 | 0.100 | -0.003 | 0.103 | 0.000 | 0.100 | -0.001 | 0.101 |
| DV8 | 1.300 | 0.058 | 1.242 | 0.020 | 1.280 | 0.020 | 1.280 |
| DV9 | 3.690 | -10.422 | 14.112 | 0.000 | 3.690 | 0.001 | 3.689 |
| DV10 | 0.986 | -316.751 | 317.738 | -16.039 | 17.025 | -15.039 | 16.025 |
| DV11 | 2.775 | 1.785 | 0.990 | 0.000 | 2.775 | -0.009 | 2.784 |
| DV12 | 33.113 | -301.763 | 334.876 | 7.160 | 25.953 | 8.160 | 24.953 |
| DV13 | 0.792 | 0.452 | 0.340 | 0.000 | 0.792 | 0.001 | 0.791 |
| DV14 | 3.989 | -0.975 | 4.964 | 0.016 | 3.973 | 0.017 | 3.972 |
| DV15 | 0.500 | -0.066 | 0.566 | 0.000 | 0.500 | 0.000 | 0.500 |
| DV16 | 0.497 | -0.251 | 0.748 | -0.017 | 0.514 | -0.017 | 0.514 |
| DV17 | 0.840 | 0.095 | 0.744 | 0.000 | 0.840 | 0.001 | 0.839 |
| DV18 | 0.667 | -0.411 | 1.078 | 0.000 | 0.667 | 0.000 | 0.667 |
| DV19 | 0.667 | -0.222 | 0.889 | 0.000 | 0.667 | -0.002 | 0.669 |
| DV20 | 1.913 | -0.130 | 2.043 | 0.029 | 1.884 | 0.029 | 1.883 |
| DV21 | 6.552 | -1.585 | 8.137 | 0.000 | 6.552 | 0.002 | 6.550 |

both objectives, the experiments reveal that the fitness landscape is deceptive. Small local changes either make no impact on the objectives of the fitness function or high impact one of the objectives making the solution not viable. However, the results from the last experiment allowed us to surpass the baseline approach by tweaking the starting parameters.

As future work, we plan to develop other variation operators and fitness functions to optimize the solutions towards both objectives. We intend to further study the fitness landscape and analyse how we can minimize the deceptive nature of the search space. We plan on exploring both optimal and diverse solutions via novelty search for this problem with different constraints, starting objectives and more DVs. Another topic of interest to go is the re-utilization of solutions, i.e., some variables from one bridge configuration could be re-utilized to solve other bridges. Furthermore, we feel that this a problem that could be used for benchmark and we are moving efforts to provide this problem to the community.

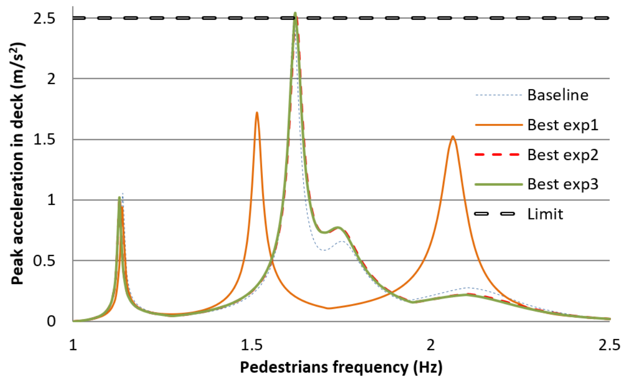


**Fig. 13.** Comparison between the dynamic responses of each of the bridge solutions. This represents the peek of acceleration of the deck peak accelerations in the vertical direction with the increase of pedestrians frequency in the bridge.

## 7 Acknowledgments

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