RECOVERING IMAGING DEVICE SENSITIVITIES: A DATA-DRIVEN APPROACH

P. Carvalho, A. Santos, P. Martins

Centre of Informatics and Systems
University of Coimbra, Polo II, 3030-290 Coimbra, Portugal
Email: {carvalho, amancio, pjmm}@dei.uc.pt

ABSTRACT

Recovering spectral sensitivities of imaging devices with indirect methods, as well as spectral stimuli estimation from device responses are ill-posed problems. All known methods have to rely on a priori information to constrain the solution space, which, in most situations, is difficult or even impossible to obtain. In this paper we introduce a simple and fully data-driven approach for indirect spectral sensitivity estimation, which does not rely on explicit *a priori* information. The method is built upon an extension of our previous work on Generalized Cross-Validation for constraint Tikhonov problems and utilizes a linear combination of band-limited basis functions.

1. INTRODUCTION

Characterization of the spectral behavior of an imaging device or pathway is important or even imperative in many image processing, analysis and computer vision problems. Several low level image processing operations, such as demosaicing [1] and color constancy [2] algorithms, require the spectral sensitivities of the imaging devices. Other common image operations, such as mapping between device dependent and independent color spaces [3][4], can largely benefit from this knowledge. High-dimensional spectral signal estimation from low dimensional device responses is another situation, with a wide range of applications in multimedia [5], industry [6][3] and medicine [7], where device spectral sensitivities have to be known.

Image sensor sensitivity characterization methods can be broadly classified into two distinct categories: (i) direct characterization methods and (ii) indirect characterization methods. In the former methods, monochromators and spectroradiometers are applied to sample the sensor's sensitivities [1]. In the latter methods, low cost color calibration charts with well known Lambertian reflectances are utilized to estimate the spectral sensitivity distribution of the sensor.

Indirect spectral characterization of an image sensor is a mathematical ill-posed problem since (i) solid state light sensors perform an input space reduction through integration over wavelengths and (ii) natural colors can be accurately approximated with just a few (typically between 3 and 9) basis functions [5]. Therefore, (i) the same device output can be produced by an infinite number of stimuli and (ii) only a limited set of linear independent data may be collected for the estimation task. To work around these problems the solution space has to be constrained. Fortunately, there are some assumptions that can be made to constrain the problem. The most commonly applied constraints are the positivity of the sensor's spectral sensitivities and the smoothness of the sensitivity function [8][9][10][5][11]. Although other types of constraints can be found in literature, positivity and smoothness are intrinsic physical properties of solid state devices. There are several alternative strategies to account for smoothness: (i) Sharma *et al.* [12] impose an upper bound on the second derivative of the solution, while (ii) other authors [10][8][9] apply a Tikhonov formulation where a regularization term is added to the object function. (iii) Hardeberg [3] utilizes a truncated pseudo-inverse to constrain smoothness. (iv) Finlayson *et al.* [5] apply a finite linear combination of band-limited basis functions to account for smoothness. All these methods require one or more user-defined parameters, being very sensible to their actual values. In [8][9] we introduced some algorithms to estimate the necessary tuning parameters from the calibration data set. These methods rely on Tikhonov formulations and extended Generalized Cross-Validation and Bayes Information Criteria to perform model identification. The devised solutions for the non-linear optimization are based on a computationally demanding genetic algorithm.

In this paper we propose a simple and efficient method for fully data-driven and indirect spectral sensitivity estimation of solid state imaging sensors, using an extension of the algorithm described in [5]. The method is built upon the identification of a set of band-limited functions and the modality, i.e., the number and wavelength location of peaks, of the device's sensitivity function $S(\lambda)$. Furthermore, all necessary knowledge, i.e., the number of basis functions and the location of sensitivity peaks, are inferred from calibration data. Although efficiency of the algorithm is not a major concern for sensor characterization, which may

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be performed off-line, it is critical for high-dimensional spectral signals estimation from low dimensional device responses. In the later case, for each cluster of similar pixels in the image a mathematically equivalent problem to sensor characterization has to be solved. Let $I(\lambda)$ be the SPD of the sensor's excitation signal and $b \propto \int I(\lambda)\, S(\lambda)\, d\lambda$ be the imaging device response (it is assumed the image has been corrected for radiometric distortions and static non-linearities). For spectral sensitivity estimation, given a set of device responses and known SPDs of excitation signals, one wants to estimate $S(\lambda)$, while for high-dimensional spectral signals estimation the goal is to estimate $I(\lambda)$ given the device's responses and sensitivities $S(\lambda)$.

The paper is organized as follows: in section 2, the algorithm is outlined. Some results obtained with this method are described in section 3. Finally, some main conclusions are presented.

2. THE DATA-DRIVEN ALGORITHM

In our method a similar formulation as in [5] is applied. Namely, using the first k standard Fourier basis, $S(\lambda)$ is described as in (1) for a sensitivity in the range $\lambda_0 \leq \lambda < \lambda_v$ ($\varpi \equiv 2\pi(\lambda - \lambda_0)/(\lambda_{v-1} - \lambda_0)$, $x_i, y_i \in \mathbb{R}$).

$$S(\lambda) = x_0 + \sum_{w=1}^{k} x_w \cos(w\varpi) + y_w \sin(w\varpi) \qquad (1)$$

Let $s_i \equiv S(\lambda_i)$, $\lambda_i = \lambda_0 + i\Delta\lambda$, $i = 0 \ldots v-1$, where $\Delta\lambda$ is the sampling interval, then s can be computed from (2), where $As = b$ ($A \in \mathbb{R}^{m \times n}$, $n = 2k+1$, $b \in \mathbb{R}^m$) are the m equations, one per calibration patch, that can be obtained from the sensor's outputs using the image formation model introduced in section 1.

$$\min\left\{\|As - b\|^2\right\} \qquad (2)$$

Besides positivity constraints, i.e., $S(\lambda) \geqslant 0$, to avoid rapid oscillations in the estimation result (see fig. 1 (left)), Finlayson *et al.* apply modality constraints. For example, a uni-modal sensitivity function with a peak at wavelength $\lambda = \lambda_p$ can be expressed as a set of linear constraints as in (3). Hence, the problem can be solved using a least square formulation subject to a set of linear inequalities $Cs \leq h$, $C \in \mathbb{R}^{q \times n}$, $h \in \mathbb{R}^q$ which account for positivity and modality of $S(\lambda)$.

$$\begin{aligned} S(\lambda_{i+1}) &\geq S(\lambda_i), i = 0, \ldots, p-1 \\ S(\lambda_{i+1}) &\leq S(\lambda_i), i = p, \ldots, v-2 \end{aligned} \qquad (3)$$

To select the number and location of peaks in the sensor curve, Finlayson *et al.* use the regression error. This selection criterion is a special case of the unbiased risk selection criterion [13], assuming calibration data without noise. Therefore, this criterion can not be applied to estimate the

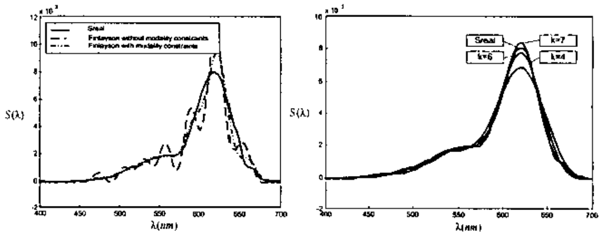

Fig. 1. (Left): Sensitivity estimation results using Finlayson's method with and without modality constraints ($k = 11$). (Right): Estimation results using Finlayson's method for different model orders.

best order k of the model in (2). Finlayson *et al.* suggest that most solid state sensors sensitivities can be adequately modelled using a number of basis functions between 9 ($k = 4$) and 15 ($k = 7$). However, as can be observed in fig. 1 (right), results can differ significantly for this range of model orders. To select the adequate model order, the function peak number and their wavelength locations, we apply an extension to the Generalized Cross-Validation criterion for inequality constraint problems (GCV-IC) [9], which estimates the generalization error of a particular solution obtained under a least squares framework subject to linear constraints. Let $C^*s = h^*$ be the set of active constraints of the solution obtained with (2) subject to $Cs \leq h$. This solution can equivalently be obtained from (2) subject to $C^*s = h^*$. Using the results presented in [9], it can be shown that, if $C^* \in \mathbb{R}^{p \times n}$ ($p < n$) is linear independent, then the GCV-IC can be computed as in theorem 1. It should be observed that if matrix C^* is not linearly independent, i.e., if $rank(C^*) = r < p$, then the singular value decomposition (SVD) of C^* may be applied to transform C^* and h^* appropriately.

Using theorem 1, the optimal model order, number of peaks and their wavelength locations can be selected as the set of values which minimize the GCV-IC criterion.

Theorem 1 *The GCV_{IC} function for the problem $\min\left\{\|As - b\|^2\right\}$ subject to $C^*s = h^*$, such that C^* is full column or line rank, is given by (4), where $\Phi = A\Theta$, $\Theta = Z_2 W_2$, $A = W_A \begin{pmatrix} D_A^T & 0 \end{pmatrix}^T Z$ and $C^* = W_C \begin{pmatrix} D_C & 0 \end{pmatrix} Z$ are the generalized singular value decompositions (GSVD) of A and C^*, and*

$$W_A^T = \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} \begin{matrix} \{ & p \\ \{ & n-p \\ \{ & m-n \end{matrix}, \quad Z^{-1} = \begin{pmatrix} \underbrace{Z_1}_{p} & \underbrace{Z_2}_{n-p} \end{pmatrix}$$

$$GCV_{IC} = \frac{\frac{1}{m}\|b - As\|^2}{\left(\frac{1}{m} trace(I - \Phi)\right)^2} \qquad (4)$$

Proof. Proof is straightforward by taking the results provided in our former paper [9].

2.1. Constraining the search space

In practice, if no *a priori* information exists on the spectral distribution of sensitivities of the imaging device, it is observed that a large set of plausible combinations k – *peak number* – *peak location* have to be searched. To avoid this, in this section an empirical method to reduce the search space is introduced. The method is based on theorem 2.

Theorem 2 *Let* $A \in \mathbb{R}^{m\times n}$, $D \in \mathbb{R}^{n\times n}$, *Ds be the discrete* 2^{nd} *order derivative of s and* $\ \in \mathbb{R}^{+}$. *The solution* $s(\ )$ *of the Tikhonov regularization problem* $\min\left\{\|As-b\|^2 + \ \|Ds\|^2\right\}$, *subject to positivity constraints, i.e.,* $s_i \geq 0, i = 0,...,v-1$, *can be equivalently computed for p active constraints from* $\min\left\{\left\|\widetilde{A}\widetilde{s}-b\right\|^2 + \ \left\|\widetilde{D}\widetilde{s}\right\|^2\right\}$, *where* $\widetilde{A} \in \mathbb{R}^{m\times(n-p)}$, $\widetilde{D} \in \mathbb{R}^{(n-p-1)\times(n-p)}$, *if* $m \leq n$ *and* $p < n$. *Further,* $\widetilde{s}_i(\ )$ *is obtained by (5), where* $c = U^T b$, $\widetilde{A} = U\Sigma Z$ *and* $\widetilde{D} = V\Omega Z$ *are GSVD decomposition of matrixes* $\widetilde{A}$ *and* $\widetilde{D}$. *Finally* $\Sigma = \begin{pmatrix} 0 & D_M \end{pmatrix} \in \mathbb{R}^{m\times(n-p)}$, $\Omega = \begin{pmatrix} D_B & 0 \end{pmatrix}^T \in \mathbb{R}^{(n-p+1)\times(n-p)}$, $D_M = diag(\ _1,...,\ _m)$ *and* $D_B = diag(\beta_1,...,\beta_{n-p}), 1 \geqslant \beta_1 \geqslant ... \geqslant \beta_{n-p} \geqslant 0$.

$$\widetilde{s}_i(\ ) = \sum_{j=n-p-m+1}^{n-p} \frac{\ _{j-n-p+m} Z^{-1}_{i,j} c_j}{\ _{j-n-p+m} + \ \beta_j^2}, i = 1,...,n-p \tag{5}$$

Proof. The first part immediately follows using permutations of columns in A, D and C^*. The second part of this result can be obtained with simple algebraic manipulations of the reduced problem.

From (5) it is seen that for large values of regularization gain , the solution $\widetilde{s}_i(\ )$ will be dominated by the linear combination of a small set of terms, those where $\beta_j \approx 1$. The influence of these terms will be persistent and, therefore, $\widetilde{s}_i(\ )$ will tend to decrease/increase monotonically as increases. Under these circumstances, it is observed that if $\widetilde{s}_i(\ )$ is a local maxima, then $\widetilde{s}(\ + \Delta\ ), \Delta\ > 0$, will also tend to be a local maxima. From this observation, the strategy for constraining the search space is straightforward to define: compute $s(\ )$ using a set of regularization gains $\ _1 < \ < \ _t$, such that $\widetilde{s}(\ _l)$ is a smooth solution (typically a set of 4 to 10 distinct). From these solutions persistent and pronounced peaks can be easily identified to formulate the modality constraints. For a fully automatic algorithm a clustering technique based on an oscillation measure $O(\ )$ (e.g. the second derivative) of the solution may be applied to compute the range of necessary regularization gains (in this case the smallest applied should lead to a highly oscillating solution, to enable the identification of at least 2 clusters). In our implementation of the algorithm, for each identified peak at wavelength λ_p, the search space is limited to $\lambda_p \pm g\Delta\lambda$, $g = 0, 1, 2$. As for the search space of the model order, we apply $k = 2, ..., 10$, which includes the search interval defined by Finlayson *et al.*

Ch.	λ_p^{real}	$\widehat{\lambda}_p$	k	$\widehat{k}$	SSE
Red	$622nm$	$624nm$	6	7	$3.75E-6$
Green	$580nm$	$578nm$	6	4	$7.37E-7$
Blue	$460nm$ $606nm$	$458nm$ $606nm$	7	7	$1.48E-6$

Table 1. Results.

3. RESULTS AND CONCLUSIONS

In this section some results obtained with the proposed method are introduced and discussed. These results (see fig. 2) are for the spectral sensitivity curves from a Kodak DCS200 camera. This camera exhibits very dissimilar sensitivity functions and, therefore, enables the evaluation of the method's performance for curves with different smoothness and modality. In these tests 24 ($m = 24$) patches of the MacBeth-Color Checker map were applied, the sampling step was fixed to $\Delta\lambda = 2nm$, $\lambda_0 = 400nm$, $\lambda_v = 700nm$ ($v = 150$) and the SSE (Sum Squared Error) values were computed by $SSE = \left\|s^{\text{real}} - s\right\|^2$ (s^{real} represents the real function obtained with the direct calibration procedure described in [1]). The obtained results are summarized in table 1. In this table the real peak location is represented by λ_p^{real} and the estimated location by $\widehat{\lambda}_p$. Column with legend k represents the ideal model order, i.e., the model order that minimizes the SSE, and $\widehat{k}$ stands for the estimated model order using the algorithm.

As can be observed the $GCV - IC$ technique enables the estimation of suboptimal solutions in the vicinity of the global optimums. This is in accordance with Craven and Wahba's theorem 4.2 [13], since the global optimum is not achievable, given that the "expectation efficiency" is usually less than 1. From table 1, for the green channel a large difference between the ideal and the estimated model order is observed. This is due to the fact that, for this channel, the sensitivities may be appropriately captured with a large range of model orders, as can be inferred from the SSE values presented in fig. 3. In fact, for $k \in \{4, 5, 6, 7\}$, very similar SSE values are achieved, and therefore the GCV criterion favors the model with less complexity (fig. 3 right), since it approximates the expected risk which is a function of the SSE and the model's complexity [13], i.e., its order for linear models.

In this paper a data-driven spectral sensitivity recovery technique for solid state imaging sensors is introduced. The method is based on an extended generalized cross-validation for constraint problems measure. No specific knowledge on the sensor characteristics is required, since

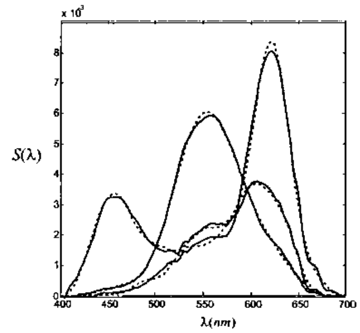

Fig. 2. Recovered RGB sensitivities of a Kodak DCS200 camera. (Continuous curves): real functions. (Dashed curves): estimated values.

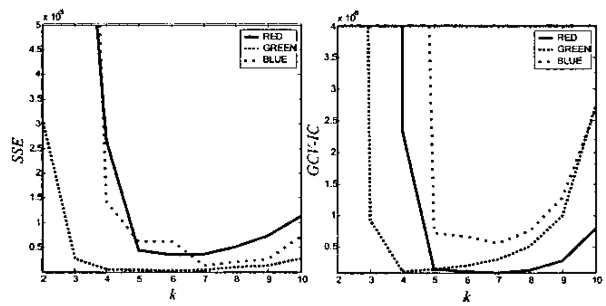

Fig. 3. SSE and GCV-IC evolution with k for the three channels of the DCS200 camera. Values are for the best selection of peaks.

the method is able to extract the needed information for constraints parametrization from the input data. This is a relevant result because, in practice, exact *a priori* knowledge is often difficult or even impossible to obtain with the required accuracy. Despite of the method's simplicity, the obtained results show that it enables the identification of suboptimal solutions in the vicinity of the global optimum, which ought to be sufficient for the most demanding color image processing and analysis applications. The main advantage of this method with respect to the algorithms described in [8][9] is its lower computational complexity. In [8][9] due to the applied Tikhnov formulation matrix $A \in \mathbb{R}^{(m+v-2)\times v}$ and $C \in \mathbb{R}^{q\times v}$, whereas in this formulation $A \in \mathbb{R}^{m\times(2k+1)}$ and $C \in \mathbb{R}^{q\times(2k+1)}$ ($v \gg 2k+1$). Further, a much more confined search space exists in the proposed method and, therefore, fewer iterations are necessary to compute the solution. Although efficiency of the algorithm is not a major concern for sensor sensitivity characterization, it is critical for high-dimensional spectral signals estimation from low dimensional device responses. Its extension for this propose may be the next step in this research.

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